Reduced Complexity Strategies for Modelling Urban Floodplain Inundation

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Abstract

Significant advances in flood inundation modelling have been achieved in the last decade through the use of a new generation of 2d hydraulic numerical models. These offer the potential to predict the local pattern and timing of flood depth and velocity, enabling informed flood risk zoning and improved emergency planning. With the availability of high resolution DEMs derived from airborne lidar, these models could, theoretically, now be routinely parameterized to incorporate the topographic complexity of urban areas, offering the potential to represent flows at the scale of individual buildings. Currently, however, computational constraints on conventional finite element and volume codes typically require model discretization at scales well below those achievable with lidar and are thus unable to make optimal use of this emerging data stream.

In this paper we review two strategies that attempt to address this mismatch between model and data resolution in an effort to improve urban flood forecasts. The first of these strives for a solution by simplifying the mathematical formulation of the numerical model by using a computationally efficient 2d raster storage cell approach coupled to a 1d channel model. This parsimonious model structure enables simulations over large model domains offering the opportunity to employ a topographic discretization strategy which explicitly represents the built environment. The second approach seeks to further reduce the computational overhead of this raster method by employing a subgrid parameterization to represent the effect of buildings and micro-relief on flow pathways and floodplain storage. This multi-scale methodology enables highly efficient model applications at coarse spatial resolutions whilst retaining information about the complex geometry of the built environment.

These two strategies are evaluated through numerical experiments designed to reconstruct a flood in the small town of Linton in southern England, which occurred in response to a 1 in 250 year rainfall event in October 2001. Results from both approaches are encouraging, with the spatial pattern of inundation and flood wave propagation matching observations well. Both show significant advantages over a coarse resolution model without subgrid parameterisation, particularly in terms of their ability to reproduce both hydrograph and inundation depth measurements simultaneously, without need for recalibration. The subgrid parameterization is shown to achieve this without contributing significant computational complexity and reduces model run-times by an order of magnitude.

Keywords: Flood inundation modelling, floodplain, reduced complexity, high resolution, topography, LIDAR, porosity, subgrid.

Introduction

Significant advances in floodplain inundation modelling have recently been achieved by directly coupling efficient 1-dimensional channel hydraulic models to a raster storage cell approximation for floodplain flows (e.g., Bates and De Roo, 2000). The strengths of this reduced-complexity model structure derive from its explicit dependence on a regular gridded digital elevation model (DEM) to parameterize flows through riparian areas. This approach offers order of magnitude gains in computational efficiency over more complex finite element and volume codes, and so enables a more critical examination of parameter and structural model sensitivities and predictive uncertainty using Monte Carlo methods (Aronica, et al., 2002).

Hitherto, applications of this framework have typically employed grid resolutions between 25-250 m with the aim of simulating flood wave propagation and inundation over long $(10^{1}-10^{2} \text{ km})$ river reaches (e.g., Bates and De Roo, 2000; Horritt and Bates, 2001). At this scale, medium-resolution digital elevation data are widely available from a variety of sources, including digitized contour data and aerial stereo-photogrammetry. However, the use of such coarse spatial discretizations may lead to poor model performance where detailed floodplain geometry, below the resolvable level of the model grid, has a significant effect on flow routing. For finite element models, the use of low resolution discretizations is primarily forced by computational constraints, so that practical applications are limited to the simplified boundary geometries typical of rural and semi-rural floodplains. The influence of subgrid topography and obstructions such as vegetation and sparse buildings are then treated relatively crudely by incorporating variability in weakly constrained roughness parameters (e.g., Bates et al., 1998; Horritt, 2000; Mason et al., 2003). Continuing advances in computational resources now offers the potential to apply more efficient codes, such as the coupled 1d-2d raster approach, at finer spatial scales where the geometrical complexity of the built environment may be explicitly modelled. If successful, this would enable an extension of the 2d modelling approach to urban landscapes where flood risk is most acutely realized.

Until recently, the parameterization of high resolution models at the scale of large urban floodplains was limited by a dearth of quality survey data. The advent and increasingly routine acquisition of dense, high precision topographic data through airborne lidar, acquiring observations at 1-2 m spacing with 0.1 m precision, now enables the construction of digital surface models (DSMs) at the building scale. Although such data have been used for rainfall-runoff modelling (Lane et al., 2004) and indeed, coarse scale inundation modelling with finite-element schemes (Bates et al., 2003; French, 2003; Marks and Bates, 2000; Mason et al., 2003), it has only very recently been considered for application at the smallest of grid scales, directly representing urban topography. Building on the opportunities presented by this emerging data source, this paper explores two contrasting approaches to modelling urban flood hydraulics using reduced complexity methods parameterized with airborne lidar. The two modelling methods are applied to simulate a major flood event that occurred in the small town of Linton in Cambridgeshire, southern England for which both flow records and distributed depth are

available. Although the model structures are used here within an urban flooding context, the paper demonstrates the potential for their use in a wide range of scenarios. The success achieved in modelling water movement through demanding urban terrain suggests that similar techniques will be useful in complex river topography even where there is minimal data availability. In particular, functionality at very high grid resolutions suggests that these model structures might be suitable for integration with sediment transfer models where issues of floodplain morphological change are important.

Reduced Complexity Modelling

The first model developed here uses the simple combined 1d channel and 2d raster scheme at a 2 m grid resolution, sufficient to explicitly represent buildings in the topographic boundary condition. Application of the raster method at this scale requires a number of modifications to existing approaches (e.g., Bates and De Roo, 2000), including; a new channel-floodplain cell mapping routine; revised and improved stability procedures; and adaptive timestepping. While, as this paper documents, the code can be implemented at this resolution, translation across scale nonetheless raises concerns over both the representation of the governing physics and the operational efficiency. For example, the computational gains in raster modelling arise largely by adopting a uniform flow approximation for floodplain flows in which gravity and frictional forces are assumed to dominate the momentum balance. While this may be reasonable for slow flows over smooth, low gradient rural floodplains, complex urban flows are likely to comprise unsteady and rapidly varying regions so that neglecting the pressure and inertial terms of the momentum equation may lead to erroneous flow paths, velocity and depth Furthermore, the computational efficiencies gained by the kinematic distributions. scheme are, to some extent, compromised by employing a regular gridded discretization over large model domains which incorporate significant inactive areas of model space and may contain upwards of 10^{6} - 10^{7} cells, requiring memory and CPU resources at the limit of desktop computers.

Sub-Grid Scale Modelling

Ironically, these concerns reflect a reversal in the historic imbalance between the physical and empirical resources available to inform numerical modelling, now evident in many aspects of hydrology. Until the advent of survey technologies such as lidar, computational flood hydraulics was increasingly limited by the data available to parameterize topographic boundary conditions rather than the sophistication of model physics and numerical methods (e.g., shock and free surface solvers). New distributed data streams such as lidar, now pose the opposite problem of how to optimally use their vast information content within a computationally realisable context. An increasingly popular approach to manage this scale problem has been sought through the development of subgrid parameterization methods. These have been applied in various forms in hydraulic modelling, for example to better resolve the inundation edge in partially-wet cells (Bates and Hervouet, 1999; Defina et al., 1994; Hervouet and Janin, 1994); to model the friction coefficient of vegetation (Mason et al., 2003); or as part of a more complex scheme to modify the full shallow water equations to account for small-scale ground irregularities (Defina, 2000).

Following this approach, the second model developed here uses dense (2 m) urban surface models to parameterize a subgrid model of the flow boundary geometry. When

embedded within a coarse (10 m) resolution model this is designed to reflect the first order influences on flow conveyance due to 'blocking' by buildings and microtopography, reducing floodplain storage and realigning flowpaths. This is achieved by determining a depth-dependent 'porosity' function on a grid-by-grid basis which varies from unity for flat open cells, to zero for cells entirely blocked by a flow obstacle. Intermediate between these, the 'porosity' reflects the difference in assumed planform storage volume and the actual available volume after accounting for subgrid features. This is then used to adjust the continuity equation and update estimates of flow depth and cross-sectional area. This scheme adds little by way of computational overheads and enables simulations at more rapidly realized coarser resolutions.

Models and Methods

The Raster Storage Cell Approximation

The basic reduced complexity model used here couples a 1d model of channel hydraulics with a 2d raster scheme for floodplain flows. The channel model uses the kinematic approximation to the Saint-Venant equations in which flows are governed by opposing gravitational and frictional forces. The coupled mass-momentum continuity equations cannot be solved analytically when overbank flow occurs and a explicit numerical solution is found by applying the finite difference form of the equations (Chow et al., 1988). The floodplain model uses a raster cell approach that has been popularized by Bates and De Roo (2000) and De Roo et al. (2000) in their model LISFLOOD-FP. Similar forms of the storage cell approach have been widely used in the past (e.g., Estrela and Quintas, 1994; Romanowicz et al., 1996) all following the blueprint of Cunge et al., (1976). This type of model has proved useful in practical scenarios (for example land-use change predictions De Roo et al., 2001; De Roo et al., 2003) and is particularly prized for its ability to produce spatial predictions with predictive performance similar to finite element codes but with much reduced run-times (Aronica et al., 2002; Horritt and Bates, 2001).

Floodplain flows are modelled by solving a continuity equation such that flow across cell boundaries is related to the volume stored in the cell and a kinematic momentum equation based on Manning's Law, to calculate flow rate:

$$\frac{\partial h^{i,j}}{\partial t} = \frac{Q_x^{i-1,j} - Q_x^{i,j} + Q_y^{i,j-1} - Q_y^{i,j}}{\Delta x \Delta y}$$
(1a)

$$Q_x^{i,j} = \frac{h_{flow}^{5/3}}{n} \left(\frac{h^{i-1,j} - h^{i,j}}{\Delta x} \right) \Delta y$$
(1b)

where $h^{i,j}$ is water depth at cell i,j, h_{flow} is free water depth between two cells, Δx and Δy are the cell dimensions, *n* is Manning's friction coefficient, and Q_x and Q_y are the flow rates in two directions between cells. The constant 5/3 is calculated by approximating channel shape as a wide, shallow rectangle.

Model Function and Stability

When flow between floodplain cells is controlled only by the momentum equation given above, instabilities occur in areas of high water depth. Several solutions have been proposed to this problem. In the version of LISFLOOD-FP described by Bates and De Roo (2000), a flow limiter was implemented to dampen oscillations between neighbouring cells which give rise to classic checkerboard instabilities:

$$Q_x^{i,j} = \min\left\{Q_x^{i,j}, \frac{\Delta x \Delta y \left(h^{i,j} - h^{i-1,j}\right)}{8\Delta t}\right\}$$
(2)

Application of a flow limiter has, however, been shown to give rise to auxiliary complications; underpredicting the advance of the inundation front and the flood volume

(Hunter et al., 2004). As an alternative, Bradbrook et al., (2004) advocated fixing the forward solution by a Courant-Friedrichs-Levy (CFL) condition calculated for flows over the entire floodplain. In a similar vein, Hunter et al., (2004) proposed an unconditional stability procedure based on dynamic timestepping defined by a stability analysis of the simplified continuity and momentum equations. This method was shown to greatly improve wetting and drying, but resulted in extremely small timestep; a condition which could prove a severe complication for the high resolution applications proposed here.

An alternative solution is therefore proposed here, which aims to improve the behaviour of the inundation front without sacrificing computational efficiency. This is implemented through a combination of several approaches. First and most significantly the basic flow limiter was extended to account for multiple outflows to neighbouring cells as dependent processes. By coupling flow directions, the limiter responds to the interaction of flow paths by proportionally scaling flows in each direction. The cell outflow limit is calculated such that in no direction will the flow be reversed in the following timestep, subject to a modification introduced to avoid neighbouring cells in equilibrium preventing flow in a perpendicular direction:

$$Q_{x}^{i,j} = \min\left\{Q_{x}^{i,j}, \left(\frac{Q_{x}^{i,j}}{Q_{x}^{i,j}+Q_{x}^{l-1,j}+Q_{y}^{i,j}+Q_{y}^{1,j-1}}\right) \cdot \left(\frac{\min\left\{H_{x}^{i,j}, H_{x}^{i-1,j}, H_{y}^{i,j}, H_{y}^{i-1,j}\right\}\Delta \Delta \Delta y}{1+1/(Number of Outflows)}\right) \left(\frac{Q_{x}^{i,j}, h^{i,j} \cdot \Delta x \Delta y}{(Q_{x}^{i,j}+Q_{x}^{i-1,j}+Q_{y}^{i,j}+Q_{y}^{1,j-1}) \cdot \delta t}\right)\right\}$$
(3)
Proportion of flow Max total outflow before surface Outflow required to empty cell

where $H_x^{i,j,}$ = water surface elevation change between neighbouring cells in x-direction. While the limiter provides a first order check on stability, it is important not to force stability when inappropriately long timesteps are used, as this could lead to significant errors in process representation e.g. retardation of wetting front. A complementary measure is therefore used to replace the use of arbitrary, global fixed timesteps as described by Bates and De Roo (2000). This involves the specification of domain-wide adaptive timestep calculated by applying the CFL condition to the 1d kinematic wave propagation for in-channel flows. This does not ensure unconditional stability (i.e., in the case where floodplain flow celerity might exceed that of the channel peak), but by linking the timestep to channel flows, it provides major computational efficiencies by avoiding a search of the entire floodplain. This represents a significant saving when the model is applied to large model domains and improves stability during the flood peak by constraining the timestep to the region of typically highest velocities but allows for extended timesteps during low flow velocities. Figure 1 illustrates the typical pattern of changing timestep obtained through this simple procedure in a flood hydrograph.



Figure 1: Adaptive timestepping through a flood hydrograph simulation by applying the CFL condition to channel flows alone.

The final improvement implemented here is to the treatment of drying cells. In the original LISFLOOD-FP code, water has been observed to continue to advance rather than recede during the drying phase (Hunter et al., 2004). To counter this we impose the start of a drying phase on edge cells, identified as those where water depth is less than a threshold of 0.01 m, and which have outflow paths but no inflow.

The steps described above attempt to enforce stability in highly simplified algorithm which is sensitive to numerical errors. While such checks are designed to suppress only numerical artefacts, in practice it is clear that the choice of stability procedure has a major influence on key model behaviour, such as the form and extent of the wetting front. Trials carried out using alternative forms of the flow limiter reveal a pattern of model sensitivity akin to an additional model parameter; capable of reducing or increasing the flood envelope. Different methods of coupling the perpendicular flows also had the potential to change the flow paths predicted by the model. This extra 'parameter' additionally interacted with the other model parameters; thus if the limiter was changed, the channel friction value required for optimum predictions might also need to be changed. Although the description of numerical methods such as flow limiters, often goes unreported, these results highlight their fundamental roles in the model.

High Resolution Application

High resolution of application of the raster method also necessitates an updating of the relationship between the channel and floodplain cells. Previous studies have generally used cell sizes similar to the channel width allowing a simple definition of the channel as a chain of cells. In the extreme case where channels narrower than the cell spacing, the 'Near Channel Floodplain Storage' model of Horritt and Bates, (2001) offers a potential

solution. However, a high resolution application to a reach with a narrow channel brings the alternative possibility of channel widths significantly greater than the raster resolution. To manage this, we propose a simple geometrical method to link the channel hydraulics to the floodplain. In this, the channel is defined by a matrix of cells, with channel cell mapped to a centreline position, according to its perpendicular distance (Figure 2). The 1d hydraulics are then solved for the centreline cells, but flows on to and from the floodplain only occur through the remapped edge cells. The rate of flow is in turn defined by comparing the channel water surface elevation to the bankfull depth for these remapped edge cells. To test the sensitivity of the model to coupling method, the results were compared, in terms of evolving channel breach points and inundation results, with those of a model using an alternative scheme in which the channel is confined within a single grid cell (McMillan, 2006). The two methods were found to be consistent.



Figure 2: Mapping channel-floodplain interface cells to channel centrelines. The arrows represent modelled flow pathways between channel edge and floodplain cells.

Sub-Grid Scale Porosity Treatment

The extensions to existing raster models described above have been implemented in order to enable computationally efficient simulations capable of modelling flow on high resolution urban DSMs. As introduced earlier, an alternative formulation to address this problem could be available through the development of a subgrid parameterization, which uses information beyond the scale of the model grid to effect more complex and non-linear controls on flow routing. Figure 3 attempts to illustrate this trans-scale approach, by revealing the subgrid complexity that a 2 m lidar-based DSM could shed on a more typical rasterization based on a 10 m bare earth digital elevation model .



Figure 3: Schematic representation of micro-topography within coarse model grids, here showing a 1 m resolution DEM embedded within a 5 m resolution model cell.

Our intention here is to develop a simple scheme which accounts for the primary subgrid influences on flow pathways through coarse grids, without resorting to conditional analytical solutions of flow around topographical features (e.g. Bradford and Sanders, 2002; Shige-Eda and Akiyama, 2003). In this we modify only the two main controls: direction and rate of flow between cells, using a coarse 10 m bare earth model as the base discretization. As described above, the rate of flow between cells is a function of the absolute and relative depths, which could be calculated more precisely by using information on the cell microtopography. This is achieved by using a 2 m DSM to specify the lowest point in each coarse cell, at which water begins to enter the cell. Above that, a *volume-depth* relationship is created based on the percentage of the cell volume which is above ground surface and hence available for water storage. We call this percentage the cell 'porosity'; and note that it is a function of the water depth in the cell as at higher depths the blocking effect of the cell microtopography is reduced. Figure 4 illustrates the pattern of flooding in a simple porous cell with grid size 2 m and topographical information at 1 m scale.



Figure 4: Flood encroachment into a model grid with explicitly modelled micro-topography; a) 2 m model gridcell with nested 1 m micro-DEM; b) schematic representation of progressive inundation.

The *volume-depth* relationship is derived by linear interpolation at discrete depth values. As an example, the relationship for the cell shown in Figure 4 is reproduced in Figure 5. This porosity function is then calculated for each cell in turn and implemented within the code as a simple lookup table.



Figure 5: Volume-Depth and Porosity-Depth relationships for the hypothetical model cell depicted in Figure 4.

The changing volume:depth relationship as water moves between cells also requires a corresponding revision of the flow limiter described in Eq. 3, to give:

$$Q_{x}^{i,j} = \min \begin{cases} Q_{x}^{i,j}, \left(\underbrace{\frac{Q_{x}^{i,j}}{Q_{x}^{i,j} + Q_{x}^{i-1,j} + Q_{y}^{i,j} + Q_{y}^{1,j-1}}_{\text{Proportion-of} \cdot flow} \right) \cdot \left(\min_{i,j} \left\{ \frac{H_{x}^{i,j}}{\frac{1}{P_{C}} + \frac{1}{Num.Flows * P_{O}}} \right\} \Delta x \Delta y \\ \left(\underbrace{\frac{Q_{x}^{i,j} \cdot h^{i,j} \cdot \Delta x \Delta y}{(Q_{x}^{i,j} + Q_{x}^{i-1,j} + Q_{y}^{i,j} + Q_{y}^{1,j-1}) \cdot \delta t}}_{Outflow required to empty cell} \right) \end{cases}$$

$$(4)$$

where P_C and P_O are the porosities at the current depth for the central and outflow cells, and $H^{i,j}$ is the water surface height difference between the central cell and outflow cells. This first order approximation assumes that cell porosity remains constant during the timestep, i.e. that insufficient depth change occurs to significantly alter porosity.

With more accurate information on the deterministic distribution of depths with the coarse cells, the direction of flows can also be controlled by the subgrid parameterisation. Consider, for example, a sloping cell where shallow water depths allow flow over only

the lowest of the gridcell boundaries. This behaviour can be simply modelled from the subgrid topography as shown in Figure 6 by determining a *depth-cross sectional area* relationship estimated for each of the four cell boundaries. A similar *depth-wetted perimeter* relationship can then be derived and applied in Manning's equation to reflect the effect of friction at the cell boundary.



Figure 6: Modelled cross-sectional flow geometry for partially flooded cell.

Data Collection and Processing for Test Applications

Upper Granta Catchment

In order to explore these two contrasting approaches to urban flood simulation, a series of numerical experiments were undertaken based on a 2 km reach of the River Granta in Cambridgeshire, UK, which has a long history of flooding. Full details of the reach and catchment hydrology are presented in McMillan (2006). The catchment is characterized by agricultural land with gentle gradients and lies on a chalk aquifer overlain by Boulder Clay. Channel widths through the study reach are typically 5-10 m with slopes in the order of 0.5% and thus close to the limit for the application of a kinematic approximation of channel hydraulics (Woolhiser and Ligget, 1967). The study reach straddles the town of Linton which has been frequently affected by severe flooding, most recently during October 2001. In this event, flooding occurred when 90 mm of rain fell in 17 hours on to an already raised water table and caused extensive damage to 72 properties, including key historic buildings in the town centre. Estimates of the return period for this event range from 100 - 400 years (Halcrow, 2005; McMillan, 2006).

Digital Elevation Modelling

An airborne lidar survey acquired in 2000 by the UK Environment Agency was used to derive both a 2 m DSM and a 10 m bare earth DEM as the boundary conditions for the high resolution and porosity models respectively. The raw x,y,z point cloud has an estimated vertical precision of 0.1 m and an average point spacing of 1-2 m, depending on swath overlap. While the point resolution is sufficient to estimate metre resolution surface models, the complex urban landscape is not well represented by direct interpolation from the raw data due to the difficulty of differentiating permeable areas of vegetation and impermeable buildings, and the inclusion of transient objects such as cars which should be removed from the DSM. Surface models were therefore derived by first segmenting the point cloud on the basis of a local neighbourhood function to yield only low-level, ground points. These were interpolated using local cubic splines to produce 2 and 10 m resolution bare earth DEMs models. The 2 m model was then augmented by adding buildings to the surface, based on 1:2500 vector street plans taken from the UK Ordnance Survey Land-Line data set, with the building heights obtained by resampling the lidar point cloud. The vector maps were also used to extract spatially variable channel width through the model grid, while bankfull depth was interpolated from surveyed cross sections sampled at 200 m or better intervals.

The 2 m resolution of the DSM was chosen to enable accurate representation of the urban area at a practical if time-consuming resolution to simulate. The 10m 'porosity' model offers a less demanding discretization, but at the expense of detail which must be recaptured from the subgrid parameterization. As described above, the porosity parameterization is stored in the form of lookup tables, recording the depth dependent porosity and boundary hydraulics for each cell. These data are processed directly from the DSM and recorded at the following depth intervals [0, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The intervals are smaller at lower depths in order to allow sufficient representation of topographical complexity. At higher depths porosity is influenced mainly man-made structures and is less dependent on depth allowing wider

calculation intervals. This pattern is illustrated in Figure 7 which shows porosity mapped over the study area at a range of depths.



Figure 7: Porosity values over the model domain mapped at increasing depth. At low inundation levels, porosity is dominated by within cell relief, but as modelled depth increases, the spatial distribution of porosity reflects the pattern of the built environment.

Channel Boundary Flow Conditions

Flow data is available across a 10 km reach surrounding Linton, from gauges maintained by the UK Environment Agency at the towns of Linton and Babraham. Stage and discharge are recorded at a 15 minute timestep. This reach is too long to be modelled efficiently using the 2 m model. To overcome this we use a one-way nesting of models, in which a low-resolution model is used at the reach scale to produce channel flows to serve as boundary conditions for a localised application of the high-resolution modelled centred on Linton itself. Nesting models in this way is common practice where localized predictions need to be contextualized, and is frequently applied in catchment and regional climate modelling (e.g., Wood et al., 2004; Kleinn et al., 2005; Cocke and LaRow, 2000; Leung and Ghan, 1999).

The value of the roughness parameters for channel and floodplain areas are not known and these were therefore treated as calibration coefficients. The reach-scale model was calibrated with respect to the downstream hydrograph only; this is considered sufficient as the only output required from this model application is the channel flow at the upper and lower point of the urban area. The model exhibited no sensitivity to floodplain friction, a pattern previously noted in storage cells models, and results from the use of flow limiter dominates control on floodplain flows (Romanowicz and Beven, 2003; Hunter et al., 2004; Hall et al., 2005). Calibration of the model was therefore focused on the channel friction parameter alone which revealed a single global optimum with n =0.023 (Figure 8). It was, however, noted that although the calibration provided a good estimate of peak values, the rate of rise and fall of the predicted hydrograph was too rapid. Reasons for this could include the use of the kinematic wave approximation, together with gauging errors at both upstream and downstream gauges due to out-of-bank The resulting channel flows were therefore recorded to provide boundary flows. conditions for the local, Linton, model, but will be used to assess hydrograph peak prediction only. A worthwhile extension of this approach would involve the calculation of uncertainty limits on the derived boundary conditions and subsequent propagation of this uncertainty into the calibration of the nested model.



Figure 8: Calibration of the channel friction coefficient (Manning's n, s/m^{1/3})by matching observed and modelled flood wave propagation at the wider reach scale.

Non-Channel Boundaries

Boundary conditions must also be imposed for those parts of the floodplain not designated as channel cells. Previous studies have used various methods to do this, a popular solution being the use of a no-flow (free-slip) boundary condition at all points except the prescribed channel (Vionnet et al., 2004). This can lead to pooling at the downstream boundary of the modelled area, requiring more sophisticated treatments such as the imposition of water surface elevation (Hardy et al., 2000) or model coupling to provide boundary data (Tucciarelli and Termini, 2000). In this case, we use first order extrapolation from neighbouring cells following the approach of Beffa and Connell (2001).

When the floodplain is near the headwaters of the catchment, the contribution of lateral inflow into the channel is significant compared to the channel input, even when there are no major tributaries in the reach. We cater for this source of runoff by prescribing an additional flow contribution to each channel cell. In the absence of any other information, this flux was calibrated from the volume change between upstream and downstream hydrographs and contributed evenly along the channel length.

Observed Inundation Data

In addition to gauged flow records, information on the pattern of inundation extent is ideally required to test for behavioural spatial model responses. Traditionally, such distributed information has been captured using remotely sensed data from either airborne or satellite platforms (see the review of Bates, 2004). No such data are, however available for the recent Linton flood and given the short duration of the flood hydrograph (c. 6 hours), obtaining bespoke overflights or satellite programming for floods of this

scale is impractical. As an alternative source of distributed information, a survey of riparian residents was commissioned following the flood, with the aim of acquiring data on maximum flood depth, flood duration and time to peak. This approach offers a much greater depth of information than simple flood extent but is open to exaggeration and provides only discrete, at-a-point, data only. It does however, provide an accessible option for flood assessment in small flashy catchments and has been successfully used in the past for model validation (Connell et al., 1998; Connell et al., 2001).

Model Testing and Results

Using the boundary conditions described above, three versions of the raster model were tested as simulators of the urban flooding in Linton. In addition to the 2 m DSM and porosity models, this included a further, baseline, model run at a 10 m resolution on bare earth DEM with no porosity information. The details of the model and simulation characteristics are given in Table 1.

Model Type	Discretization Scale (m)	Subgrid Parameterization	Grid Size (x,y)
Baseline	10	No	144 x 120
Porosity	10	Yes	144 x 120
Urban DSM	2	No	720 x 600

Table 1. Details of the three model structures and applications

Calibration Methodology and Model Performance Evaluation

Validation studies of floodplain inundation models have highlighted the importance of evaluating model response in terms of both the dynamics of flood wave propagation and the spatially distributed pattern of inundation. Previous studies of the raster modelling approach have identified significant equifinality (*sensu*, Beven and Binely, 1992) in model parameterizations, with distinct optima existing for each of these performance indicators, and a failure to jointly reproduce these two crucial behaviours. For this study, only one key sensitive parameter was explored, the channel friction coefficient. Following the preceding discussion, the predictive performance of the model was evaluated in terms of the flood wave dynamics (downstream hydrograph fit), the spatial pattern of inundation and additionally a Bayesian metric which combines these two characteristics.

a) Calibration and performance evaluation by inundation extent and depth

Conventionally, the predictive accuracy of inundation extent is measured through a comparison of mapped and modelled flood boundaries (e.g., Bates and De Roo, 2000; Bradbrook et al., 2004). This is, in effect, a binary test (wet vs dry) rather than a continuous assessment of predicted depth. In this study, such a binary analysis could be undertaken by comparing houses reported to have flooded against the simulations. However, because the observed database is limited to those properties which actually flooded, this would lead to a maximally wet simulation being preferred. Instead, it is possible to take advantage of the extra information gathered during the residents' survey by validating on the depth of flooding as well as extent. Although, this could be achieved using a traditional least squares test, it was felt important to recognize the questionnaire responses were likely to incorporate potential errors due to poor estimation/memory of actual flood depths. A possible technique designed to incorporate such uncertainty is available through the use of a fuzzy goodness of fit measure (Freer et al., 2004; Seibert

and McDonnell, 2002). In this case a simple fuzzy scoring system was used, in which each observation (or building in the database, where n = 38) is given a score between 0 and 1 depending on quality of fit between the simulated and reported maximum flood depth. A score of unity indicates a 'perfect' fit and is attributed when the 'observed' and simulated depth correspond to within 0.1 m. The score then decreases linearly to zero when the discrepancy is equal or greater than 0.5 m. A global statistic for the simulation is then calculated as the arithmetic average of the individual property scores.

b) Calibration and performance evaluation by floodwave translation

The Nash-Sutcliffe R-square measure was initially considered for measuring the downstream hydrograph fit, however it was found to be unable to adequately differentiate hydrographs based on a single storm. Instead, benefiting from the observation that the simulated hydrographs typically took a consistent shape, a vector measurement of peak offset is used. This combines attenuation and lag errors to give an intuitive goodness-of-fit measure.

c) Multi-criteria Calibration

In this study, we have attempted to combine the two above model responses into a single metric. This helps to clarify whether simulations are 'getting the right answers, for the right reasons', correctly predicting the spatial and temporal pattern of floodplain storage and release which determines the downstream hydrograph attenuation and lag. Any dual calibration is likely to be a trade-off between different responses, as demonstrated by the 'Pareto Set' concept of Gupta et al., (1998), but can in some cases improve the identifiability of model parameters (Kuczera and Mroczkowski, 1998). To try and make some comparison between different members of the Pareto Set, one option is to combine the validation criteria into a single goodness-of-fit index (Beven, 1993; Beven, 2000) for example using Bayesian updating, weighted mean or fuzzy set operations. Here we use a linear mapping of the response spaces for a comparison of available parameter sets with hydrograph and inundation data given equal weight. The general formula for k criteria is shown below although only a two-criterion case is applied here (the flood propagation and inundation scores respectively):

$$L(\Theta|\underline{Y}_{1}\dots\underline{Y}_{k}) = \frac{1}{k} \cdot \sum_{k} \frac{L(\Theta|\underline{Y}_{k}) - \min_{i,j} L(\Theta_{i}|\underline{Y}_{j})}{\max_{i,j} L(\Theta_{i}|\underline{Y}_{j}) - \min_{i,j} L(\Theta_{i}|\underline{Y}_{j})}$$
(4)

with data sets \underline{Y}_k , parameter sets Θ , likelihood L().

Results

a) Calibration and performance evaluation by Pattern and Extent of Inundation and Flood Depth

Model simulations were carried out for the channel friction coefficient, Manning's n, over the range 0.02-0.06 s/m^{1/3}. Plots showing the maximum flood extent for each case are shown in Figure 9. Although differences in inundation patterns can be seen between the three model types, in many places the envelopes seem broadly similar. Particularly for the higher values of the channel friction parameter, this reflects the relatively steep topography at the boundary of the natural floodplain, which serves to constrain the flood water. In order to differentiate model performance, the objective measures described in the previous section are therefore used.



Figure 9: Floodplain Inundation in the urban model domain around Linton. Figures A, B and C show results for a range of channel Manning's n values: (a) 2 m (b) 10 m with Porosity (c) 10 m

A preliminary insight into the comparative performance across model type and channel friction parameter is revealed by comparing 'observed' and modelled flood depths at the inundated buildings as shown in Figure 10.



Figure 10. Observed and predicted maximum flood depths in reporting households, conducted for channel Manning's n between 0.03-0.06. Results are presented for the three model structures; a) High resolution 2 m model; b) 10 m model with porosity treatment; c) 10 m model without porosity

Visual inspection shows that, although in all cases there are high levels of error in the model predictions, the optimal results are obtained for values of n between 0.05-0.06, higher than field observations for a fine gravel, low sinuosity, relatively clean channel might suggest (e.g., Chow et al., 1988). This likely reflects a combination of factors including error in upstream hydrograph record, alternative flow sources such as overland flow, groundwater breach of cellars, as well as process representation errors which lead to channel roughness compensating for unmodelled processes. First impressions also suggest that the models run at 2 m and 10 m with porosity information are less prone to outliers than the baseline 10 m model. A structured comparison is obtained through the fuzzy inundation score plotted in Figure 11.



Figure 11: Fuzzy validation score for the three model structures varying with channel roughness.

This comparison shows that fit improves with increasing channel roughness for all three models, with the performance of the baseline 10 m model falling below the two other approaches. This drop in performance is due to overflooding in some areas, demonstrated in the outlying points in Figure 12. The houses represented by these outliers are circled and clearly highlight the wider flood envelope predicted by the baseline model compared to the porosity model. This wider and smoother flood outline reflects the increased volumes of water on the floodplain predicted by the baseline model, which fails to account for the boundary detail incorporated by the subgrid porosity treatment subtly controls flow pathways.



Figure 12: Maximum flood envelopes predicted for the two 10 m models, with and without the porosity treatment. The circle indicates properties where depth is severely overestimated in this case.

b) Calibration and performance evaluation by Downstream Hydrograph

Figure 13 shows simulated downstream hydrographs for the three models with channel roughness values varying from 0.02-0.06 s/m^{1/3}. In each case, the downstream boundary condition derived from the reach-scale model discussed above is plotted for comparison. The optimum value the friction parameter clearly varies with model structure, but is always in the range 0.02-0.04 s/m^{1/3}. At higher values of n, all models overestimate the lag time, although the 2 m and porosity models achieve a reasonable representation of hydrograph shape and magnitude while the baseline model significantly overestimates attenuation in the reach.



Figure 13: Flood wave routing performance for the three model structures, shown for roughness coefficients varying between 0.02-0.06. A similar pattern of performance is found for the both the 2 m and 10 m with porosity structures across a wide range of roughness values, but excessive attenuation as more water is routed on the floodplain in the coarse 10 m structure.

The hydrograph validation scores are plotted in Figure 14 which clearly emphasizes the superior performance of 2 m and porosity models which achieve high scores across a wide range of channel friction coefficients while the baseline model has a much restricted behaviour parameter space.



Figure 14. Validation scores calculated using the combinatorial Peak/Lag statistic

The over-attenuation evident in the baseline model indicates an excess of water being routed on to the floodplain, as evidenced by the spatial model responses discussed above. This again emphasizes the importance of the porosity information in restricting and directing flow. An examination of the evolving channel breach points showed that these were very similar with or without porosity information, however the volumes able to flow through the breach points were much greater when porosity information was not used. Figure 15 shows typical differences in flood volumes when porosity information is introduced into the model, through a series of snapshots of flood depth during rising limb, flood and recession.



Figure 15: Snapshots of inundation patterns predicted by the 10 m codes, without (a) and with (b) the porosity treatment. The 'blocking' effect of the buildings is evidenced in the discontinuous and irregular pattern of floodplain depths. Inundation depths are references to the attached greyscale legend.

c) Multi-criteria Calibration

The combinatorial likelihood measure, incorporating both aspects of predictive performance is plotted against the channel roughness parameter for each of the three models in Figure 16.



Figure 16. Validation scores calculated for the combinatorial inundation and routing statistic

This shows the 2 m and porosity models to give similarly good fits to recorded inundation levels and flood routing using channel roughness of around 0.05 s/m^{1/3}, while the baseline model fails to achieve equivalent scores and peaks at a roughness of 0.03 s/m^{1/3}. This reflects the inability of the 10 m model to produce realistic predictions for both downstream hydrograph and inundation extent for the same value of the channel friction parameter.

Discussion

This case study has added further weight to the case for including porosity information as a viable method for utilizing high-resolution topographic data in inundation modelling (previous exponents include Hervouet *et al.*, 2000; Yu and Lane, 2006b). In tests against the baseline model, both the full 2 m DSM and porosity parameterizations, were found to behave similarly, and with greater predictive power and robustness. The downstream hydrographs derived from the two models were almost identical and the distributions of depths also found to be alike. The main differences observed were for low values of channel friction where the porosity predicted lower water depths.

Given the superior performance of both the DSM and porosity models, some joint conclusions can be drawn. First, the distribution of overbank depths highlighted the tendency of the baseline model to overflood. Had a simple binary analysis of the flooded envelope been undertaken, this result would have been latent, as the flood outlines are less easily distinguished and the over-prediction of flood depths is highly localized. Bates and De Roo (2000) observed similar behaviour in their model LISFLOOD_FP, when simulating floods on the River Meuse at grid resolutions between 25-100 m. They found that at the lowest resolution, depths were significantly overestimated and inferred this to result from smoothing out levee structures which allowed more water onto the floodplain. A similar finding has been outlined by Yu and Lane (2006) who also found flood extent and depth to scale with grid size. In this study, depth overestimation was found to particularly affect houses near to the inundation boundary. This adds increased weight to the suggestion by Hunter et al. (2004) that model conditioning on flood outline data alone is an insufficient measure of predictive performance.

Improvements in inundation prediction are linked directly to flood routing behaviour, as the impeded inundation on the floodplain prevents excess water leaving the channel and therefore controls the 1d channel hydraulics. This pattern is neatly revealed by the multicriteria validation, which suggests that different model implementations may require a substantially different roughness to provide their optimal performance: the baseline model peaks at $n = 0.03 \text{ s/m}^{1/3}$ while the high-resolution models peak at $n = 0.05 \text{ s/m}^{1/3}$. This sensitivity to grid size and the need for subsequent re-calibration has been recognized in several other studies and in addition to the influence on flow blocking discussed above, has also been attributed to increased in channel flow velocities in fine grid models (Connell et al., 1998; Nicholas and Mitchell, 2003).

In the light of these findings it is reasonable to compare the 'best' results in terms of goodness-of-fit for the three implementations and recognize that roughness coefficients are not necessarily transferable across scales. They should rather be considered as 'effective' parameters that are dependent on grid resolution rather than absolute quantities. Additionally, it is important to note that they may also interact with other model processes and their individual role cannot easily be disentangled. However, it remains clear that by including 'excess' topographic information, predictive performance can be significantly enhanced and indicates simultaneously reliable predictions of flood routing and floodplain inundation may be possible.

This helps to explain the similarity of results between the 2 m and porosity models. The improvement in performance may be considered in terms of flow pathway definition. Including the subgrid porosity function allows flow volume and direction to be controlled in a manner similar to the full higher resolution implementation; a concept referred to by Lane (2005) as flow 'blockage'. Models including porosity information are therefore able to provide a much more realistic representation of flood evolution and improve simulated pattern of downstream attenuation. This is result is of practical importance, for it suggests that the coarse discretization strategies can be implemented with significant computational savings over the use of detailed DSMs. In this case, benchmarked tests on a standard desktop pc found runtimes for the porosity model to require only 1/38 of the DSM model (Table 2).

Model Type	Running Time (mins)	
10m grid	80.4	
10m with Porosity	235.0	
2m grid	9016.3	

Table 2: Execution times for the three model structures, benchmarked on a Pentium 4, 3.2 MHzPC with 1.5 Gb RAM, based on simulations with the optimal channel friction coefficient - $n = 0.05 \text{ s/m}^{1/3}$

Conclusions

This paper presents new approaches to the use of high-resolution topographic data within a 2d floodplain inundation model. Increasingly, airborne mapping of floodplain topography offers the scope to derive digital surface and elevation models at higher resolution than computational resources permit in existing conventional hydraulic FE codes. We have outlined two reduced complexity or fast, approximate, approaches to address this scale problem with the aim of improving flood forecasts in geometrically complex areas. Both of these involve modifications to the increasingly popular raster storage cell approach to flow modelling, but differ in the way they use this 'excess' terrain information to inform floodplain flowpaths. In the first approach, improved numerical stability procedures enable direct incorporation of the geometry of the built environment into the topographic discretization at very high spatial resolution. In the latter approach, this detailed DSM is instead used to provide an subgrid parameterization of the microtopography, through a depth-dependent porosity function. Both models are shown to out-perform a baseline raster model in terms of both flood routing and spatial inundation patterns. The successful application of the porosity model is particularly encouraging in that it offers scope to improve flood forecasts in urban areas by simply accounting for the effects of flow blocking on conveyance, without the need to incorporate more complex non-linear terms in the momentum budget. This approach therefore offers significant predictive gain with little computational cost.

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