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Effect of spatial variability and seasonality in soil moisture on drainage thresholds and fluxes in a conceptual hydrological model

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ABSTRACT

This paper uses soil moisture data from 17 recording sensors within the 50 km² Mahurangi catchment in New Zealand to determine how measured variability in soil moisture affects simulations of drainage in a typical lumped conceptual model. The data show that variability smoothes the simulated field capacity threshold such that a proportion of the catchment contributes to drainage even when mean soil moisture content is well below field capacity. Spatial variability in soil moisture controls by extension the catchment drainage behaviour: the resulting smoothed shape of the catchment-scale drainage function is demonstrated, and is also determined theoretically under simplifying assumptions. The smoothing effect increases the total simulated discharge by 130%. The analysis explains previous findings that different drainage equations are required at point-scale vs catchment-scale in the Mahurangi. The spatial variability and hence the emergent drainage behaviour are found to vary with season, suggesting that time-varying parameters would be warranted to simulate drainage.

INTRODUCTION

Soil moisture is an important control on runoff production in a catchment, through its effect on evapotranspiration and drainage. Conceptual hydrological models use soil moisture storage estimates to simulate modelled fluxes, typically operating at the catchment or sub-catchment scale without explicitly representing variability below that scale. This level of spatial detail is often commensurate with the availability of flow data for calibration or validation and models can perform well in simulating both measured flows and soil moisture values (e.g. Calder et al., 1983).

However, it is well known that soil moisture is heterogeneous in a catchment and depends on many factors including local climate, topography, soil type and depth, aspect and vegetation (e.g. Brocca et al., 2007; Crave and GascuelOdoux, 1997; Kim et al., 2007; Nyberg, 1996; Penca et al., 2009; Western et al., 1999). Therefore, fluxes controlled by soil moisture will also be heterogeneous, causing integrated catchment fluxes to differ in form from point fluxes: the ‘scaling problem’ (Blöschl and Sivapalan, 1995; Bronstert and Bardossy, 1999). Changes in catchment runoff with scale can also be caused by emergent processes dependent on spatial patterns, such as connectivity of transient saturated areas (e.g. Lehmann et al., 2007). The effect of variability in soil moisture on runoff production has previously been addressed using a distribution of soil moisture storage capacities, notably in the PDM model (Moore, 2007), and similarly in the VIC (Variable Infiltration Capacity)/ARNO/Xinanjiang model (Todini, 1996; Wood et al., 1992; Zhao and Liu, 1995).
In this scientific briefing we use a dense spatial/temporal dataset of soil moisture content in the Mahurangi catchment, New Zealand, to show explicitly how heterogeneity in soil moisture affects simulations of drainage in a typical lumped conceptual model. Research has identified deep drainage as a dominant flow pathway, even in small catchments (Bestland et al., 2009; Graham et al., 2010; Sklash et al., 1976). We discuss the causes of spatial variability in soil moisture, and hence drainage; which go beyond storage capacity to include factors as listed above. The degree of variability may therefore vary over time: we consider the implications for seasonal variability in the nonlinearity of drainage, and demonstrate the effects in the Mahurangi catchment.

STUDY AREA AND DATA COLLECTION

Mahurangi catchment is located in the North Island of New Zealand (Figure 1a). The climate is generally warm and humid, with mean annual rainfall of 1628 mm (with spatial range 1440 – 1830 mm) and mean annual pan evaporation of 1315 mm. Soils are silty clay loams, with smaller areas of alluvial clay-rich soil in lowland valleys. Land use is predominantly grazed pasture with small areas of shrub; there are also areas of exotic forest (*pinus radiata*) in the south and native forest in the north.

The Mahurangi River Variability Experiment (MARVEX; Woods et al., 2001) ran from 1997-2001, and investigated the space-time variability of the catchment water balance. Data was collected from recording soil moisture sensors in 18 locations, grouped into 6 sites (Figure 1b). Four sites are under pasture (Claydons, Carrons, Satellite Right, Satellite Left) and two are under exotic forest (Marine East, Marine West). At each site, sensors were arranged at three hillslope locations and two depths (the first at 0-300 mm, and the second over the 200 mm of soil at the bottom of the soil profile). Only the upper sensors are used in the analysis that follows, for simplicity, although the deep sensors behaved similarly. Measurements were made at 30 minute intervals for 29 months; the sensors used were Campbell Scientific CS615 (Frequency Domain Reflectometry) probes.

An extensive effort was made to ensure correct calibration of the soil moisture sensors by comparison with neutron probe and gravimetric measurements (Western and Seyfried, 2005; Western et al., 2004; Wilson et al., 2003). Any periods of poor quality data were removed. One site at Marine West was excluded from the analysis due to consistently poor data quality. Mean soil moisture content was calculated using only periods of time when all the remaining 17 sensors were operational (67% of the period retained; most outages occurred during the final year). Field capacity was estimated for each soil moisture series as site-specific laboratory measurements were not available, and there was too much variation to enable transfer of values between sites. The method used was to observe the point of ‘change of slope’ between fast drainage after a storm, and slower sustained drainage, which leads to an easily visually distinguishable winter equilibrium state. Such visual methods are commonly applied in order to estimate field capacity (Calder et al., 2002).

Previous research has helped to identify the dominant runoff generation mechanisms in the Mahurangi, overturning early ideas that flow paths were confined to the upper 30 – 50 cm of soil by an impeding clay layer. Western et al. (2004) showed that correlation lengths of near-surface soil moisture patterns do not change with season, suggesting instead that deeper lateral flow paths (consistent with the gradational soil profile) may control flow. Tracer studies reported by W. B. Bowden (2009, personal
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communication) concluded that deep drainage to the saturated zone was the dominant flow path from hillslopes in the Satellite subcatchment. Evidence came from experiments in which bromide, chloride and deuterated water were applied to the upper or lower hillslopes. None of the tracers were detected in streamwater at the base of the hillslope during the 2 month experiment, and the tracers often bypassed samplers within the soil matrix, presumably via preferential flow pathways.

**SPATIAL VARIABILITY IN DRAINAGE OCCURRENCE**

Drainage through an unsaturated soil matrix occurs once gravity overcomes tension forces, as described by Richard’s equation. Field capacity is defined as the threshold soil water content where flow occurs, i.e. the amount of water left in soil after excess has drained away, commonly approximated as a matric potential of 1/3 bar. As moisture content increases, another threshold, saturation point, activates additional flow paths i.e. saturation excess runoff. Variability in reaching either threshold (field capacity or saturation) could be caused by [1] variability in the storage available or [2] variability in the soil moisture content, or both. The well-known PDM model (Moore, 2007), assumes the former; i.e. direct runoff only occurs where mean soil water content exceeds storage capacity. In this paper we account for both possibilities in the context of vertical drainage; i.e. we assume that local drainage commences (or increases rapidly) when local soil moisture content exceeds local field capacity.

To completely define the effect of spatial variability on drainage occurrence it would therefore be necessary to know, for a given mean soil moisture content, the exact distributions of field capacities and soil moisture contents over the catchment. In the Mahurangi, the soil moisture data can be used to estimate this distribution, by approximating the proportion of the catchment exceeding field capacity with the proportion of soil moisture sensors exceeding field capacity. This assumes that the sensors are distributed in a way that is representative of the catchment soil properties; Woods *et al.* (2001) discuss the efforts made to achieve this based on consideration of the processes which dominate the spatial distribution of soil moisture. Important qualities were identified as divergent/convergent areas, slope, aspect, soils and vegetation, and efforts were made to sample both mean and extreme behaviour. Four pasture areas and two forest areas were chosen as being representative of the broader catchment which consists of 53% pasture, 44% forest, 3% other crops/shrubs. The rationale was based on work by Grayson and Western (1998) which indicated the likely existence of representative sites, termed Catchment Average Soil Moisture Monitoring (CASMM) sites. More generally, the method approximates upscaled behaviour in that part of the wider catchment which is represented by the sensors.

Hence, we use the soil moisture series to analyse the relationship between mean catchment soil moisture (approximated as sensor mean) and the proportion of sensors exceeding field capacity. These two quantities are calculated at each timestep, and the aggregated results are shown in Figure 2. The plot shows that spatial variability in soil moisture has smoothed the hard field capacity threshold. A model which did not include variability would simply show the proportion of the catchment exceeding field capacity switching from 0 to 1 at the mean field capacity value. Instead, the points lie on a curve and show that some parts of the catchment can rise above field capacity, and hence contribute to drainage, even when the mean soil moisture is far below the mean field capacity.
A simple assumption would be that at a given time, soil moisture in the Mahurangi was normally distributed about the mean value, with fixed variance. This would lead to a curve fitted through the points in Figure 2 having the shape of a normal CDF. Instead, the points (1) show a longer tail towards drier soil moisture values, and (2) are negatively skewed such that when mean soil moisture is at field capacity, approximately 60% of the catchment is at field capacity or wetter. These findings can be explained as follows.

(1) Previous studies have found experimentally and with theoretical analysis that spatial variability in soil moisture is greatest at mid-range values of mean catchment soil moisture, and decreases for drier or wetter conditions (e.g. Penna et al., 2009; Ryu and Famiglietti, 2005; Vereecken et al., 2007). Therefore variations from the hard field capacity threshold could be expected to be greatest for mid values of mean soil moisture (here 40-45%), causing the extended tail.

(2) Assuming that drainage increases with soil moisture content, locations with soil moisture above the mean would drain faster than those below, causing a negatively skewed distribution. This simple intuition is confirmed by studies of the effects of covariance between soil moisture and water fluxes (due to soil, vegetation, precipitation, topography, and initial moisture variability) which can act to produce or destroy variance in soil moisture over time (Albertson and Montaldo, 2003; Entekhabi and Rodriguez-Iturbe, 1994; Isham et al., 2005; Montaldo and Albertson, 2003). In the Mahurangi, the distribution shape can be approximated by the distribution of soil moisture values (Figure 3). In part, this approach ‘trades time for space’ (Loague, 1991), by using 17 sensors at multiple time steps to approximate the complete distribution.

Close to field capacity (scaled soil moisture value of 1), the distributions are negatively skewed, as hypothesised. This distribution shape is similar to, and hence provides evidence for the Pareto distribution with b<1, as commonly used in the PDM model. The degree of skew varies between sites (Figure 3b) with the forested sites (Marine East and Marine West) having greater skew than the pasture sites. Note also the double-peaked form of the histograms, suggesting two ‘preferred’ soil states, wet and dry. The same form has been observed in other soil moisture studies (e.g. Penna et al., 2009), however when considering drainage fluxes we are chiefly concerned with the distribution around field capacity.

**Spatial Variability in Drainage Quantity**

Spatial variability in soil moisture controls by extension the emergent catchment vertical drainage behaviour. The soil moisture data from Mahurangi can be used to directly quantify the effect of spatial variability on a common drainage scheme found in hydrological models:

\[
    d_t = \frac{1}{\alpha} (S_t - f_c)
\]

(Eq 1)

Where \(d_t\) is drainage at time \(t\), \(S_t\) is scaled soil moisture at time \(t\), \(f_c\) is field capacity, \(\alpha\) is a constant.

This scheme represents drainage as a linear function of soil moisture content above field capacity. The scheme was found to be an appropriate description of point-scale drainage behaviour in the Mahurangi (Clark et al., 2011; McMillan et al., 2011). It is
also found in popular models such as the Precipitation-Runoff Modeling System (PRMS; Leavesley et al., 1983), the UBC Watershed Model (Quick, 1995) and the Sacramento Model in the case of constant lower zone storage (Burnash et al., 1973).

For each time step in the soil moisture series, the soil moisture value and field capacity value at each location were used to calculate the scaled drainage at that location (on a scale of 0-1 where 1 represents maximum drainage at saturation; equivalently set \( \alpha = \) saturated moisture content \(-\) field capacity) using Eq 1. The mean of those values approximates the mean drainage over the catchment accounting for spatial variability. Figure 4 shows the values by plotting mean drainage as a function of mean soil moisture. The model behaviour not accounting for spatial variability is shown for comparison as a linear relationship between mean soil moisture and drainage, based on mean field capacity.

Figure 4 shows that when spatial variability is taken into account (shaded area), drainage always occurs at values below the mean catchment field capacity: the hard drainage threshold of the catchment-scale model has been smoothed. While the value of the drainage might seem relatively small below mean field capacity (i.e. a small smoothing effect), the effect on soil moisture dynamics can be large because the system spends the majority of time at moisture contents below field capacity. This can be demonstrated by comparing the average per-timestep drainage calculated using models including vs not including spatial variability. The average drainage is 130% greater in the model that includes spatial variability.

These results confirm that a nonlinear drainage model with a smoothed field capacity threshold is needed to simulate the effects of spatial variability of soil moisture in the Mahurangi. The finding provides an explanation for previous work in the Mahurangi (Clark et al., 2011; McMillan et al., 2011) which showed that although a linear drainage model better simulated soil moisture timeseries at the point scale, a nonlinear power law model more accurately reproduced runoff ratio and runoff timing at the catchment scale. However, the power law function leads to some (if small) drainage at all values of mean soil moisture, whereas the results from this study suggest a different formulation where drainage only occurs above some lower limit (although the limit might be smaller if more soil moisture sensors were available).

To use the results to specify an appropriate catchment-scale drainage function for the Mahurangi catchment, either an empirical function could be fitted to the data in Figure 4, or a function of similar shape could be found. One function which has similar behaviour is the logistic function used by Kavetski et al. (2006) and Clark et al. (2008) to smooth thresholds in snow melt and water storage overflow fluxes. Although in those cases the motivation was primarily to remove numerical artefacts and improve the performance of parameterisation optimisation methods, the authors note that the approach may also help provide model equivalence for spatial variation in thresholds. Alternatively, the shape of the required function, i.e. the expectation of the drainage as a function of the mean soil moisture, can be calculated directly under certain simplifying conditions. For example, assume that the distribution of local soil moisture values is Gaussian around the mean catchment soil moisture, such that variations from the mean are independent between sites.

Using Eq 1, the expectation of the drainage can be found as
\[
E(d_\ell) = \int_0^\infty D \cdot f(D) \cdot dD = \int_0^\infty D \cdot \phi(\alpha D + fc) \cdot dD 
\]  
(Eq 2)

Where \(d_\ell\) is drainage, \(\varphi\) is the normal \(N(s, \sigma^2)\) pdf with mean soil moisture \(s\) and variance \(\sigma^2\), \(fc\) is field capacity and \(1/\alpha\) is the slope of the linear drainage function. An analytic solution for this integral can be found as follows:

\[
E(d_\ell) = \frac{1}{2\alpha} \left[ \frac{2\sigma^2}{\pi} \cdot \exp\left(-\frac{(fc-s)^2}{2\sigma^2}\right) - (fc-s) \cdot \text{erfc}\left(\frac{fc-s}{\sqrt{2\sigma^2}}\right) \right] 
\]  
(Eq 3)

The shape of the function is shown in Figure 5. The shape can easily be modified according to the required assumptions, e.g. to include nonstationary variance which depends on soil moisture content. A Gaussian distribution was used here as a simple model but more complex distributions could be chosen, for example to represent negative skew in soil moisture values as suggested by Figure 2. Depending on the distribution chosen, a numeric solution for the integral may be required.

**SEASONAL VARIABILITY IN DRAINAGE**

It is useful to compare these findings with the simpler assumption that the runoff production is controlled by the distribution of soil moisture storage capacities (e.g. Moore, 2007), as opposed to distributions of both storage and soil moisture content as assumed here. Where the distribution of soil moisture content is constant with time (for given mean soil moisture), the two approaches are similar. However, under real conditions the degree of soil moisture variability has multiple causes and can vary with time, leading to a corresponding variation in the catchment-scale drainage function. This may be for multiple reasons. Any annual cycle in soil moisture may vary locally, causing increased variance during wetting-up / drying periods in spring and autumn. Spatial and temporal variability in rainfall may also change by season according to variation in weather system types. To examine the seasonal effect in the Mahurangi basin, the data were divided by season and replotted (Figure 6).

Figure 6 shows that variability in soil moisture and hence the shape of the drainage function is dependent on season. In summer and autumn little can be concluded as few sites exceed field capacity. In winter, variation about the mean field capacity threshold is approximately symmetric (Fig 6a). At this time of year soils typically fluctuate around the field capacity moisture content. In spring however, the smoothing effect of soil moisture variability is more pronounced. Drainage occurs at lower mean moisture contents and at higher rates than in winter, even for the same values of mean soil moisture. This is shown in Figures 6a and 6b where ‘Spring’ measurement points deviate further than ‘Winter’ points from the behaviour predicted by the model which does not include variability (shown by the solid blue line). Increased variation in spring may be due to local differences in the annual cycle as hypothesised above, and may contribute to the well-known difficulty in simulating catchment behaviour during drying or wetting periods (e.g. Pinol et al., 1997). This behaviour has interesting implications for conceptual hydrological model design as it indicates that additional smoothing of the drainage function in spring would more closely match measured soil moisture behaviour. For example, in the analytic solution given above (Eq 3), the variance parameter \(\sigma^2\) could be varied according to season. Although in general the need for time-varying parameters is noted as an indication of model structural flaws...
(Reichert and Mieleitner, 2009), in this case it would be justified from physical behaviour.

CONCLUSION

This paper used dense soil moisture records from the Mahurangi catchment to demonstrate how emergent drainage behaviour at the catchment scale relates to small-scale variability in soil moisture content. While the paper focussed on drainage behaviour, similar results would be expected for other model thresholds (e.g. infiltration excess dependent on rainfall rain thresholds). The analysis showed that smoothing hard thresholds in a lumped conceptual model is a necessary upscaling step and explained previous findings that different drainage equations were required at point vs catchment scale in the Mahurangi. The method is transferable to other small catchments in which deep drainage is the dominant runoff generation mechanism. Importantly, the degree of variability and hence the catchment-scale drainage behaviour varies with season. The recommended smoothed drainage functions show how point scale soil moisture data can be used for model design while explicitly recognising the scale difference involved.

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FIGURES

Figure 1. (a) Location map for Mahurangi catchment in North Island of New Zealand (b) Locations of soil moisture recording sensors within the Mahurangi catchment

Figure 2. Proportion of soil moisture sites exceeding estimated field capacity as a function of mean soil moisture % by volume across all sites. The vertical line shows the mean field capacity for all sites for comparison.

Figure 3. Histograms of soil moisture, scaled between wilting point (0) and field capacity (1), for (a) All Sensors and (b) Sensors by location.

Figure 4. Mean catchment drainage as a function of mean catchment soil moisture. The range of values caused by soil moisture variability is shown (shaded area), for comparison with the linear relationship discounting variability (line).

Figure 5. The catchment-scale drainage function relating mean soil moisture to total drainage under the simplifying assumption that soil moisture is normally distributed about the mean. Results are given for stationary soil moisture variance (blue) and nonstationary variance (red; using an example where variance increases linearly to a maximum at 45% soil moisture content, after which it decreases linearly).

Figure 6. (a) Proportion of soil moisture sites exceeding estimated field capacity as a function of mean soil moisture, plotted by season (offset for clarity). (b) Mean catchment drainage as a function of mean catchment soil moisture, plotted by season. Dotted line shows range of hidden points from Spring season.
REFERENCES


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250x139mm (300 x 300 DPI)
Figure 2. Proportion of soil moisture sites exceeding estimated field capacity as a function of mean soil moisture % by volume across all sites. The vertical line shows the mean field capacity for all sites for comparison.

85x71mm (300 x 300 DPI)
Figure 3. Histograms of soil moisture, scaled between wilting point (0) and field capacity (1), for (a) All Sensors and (b) Sensors by location.

103x83mm (300 x 300 DPI)
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85x70mm (300 x 300 DPI)
Figure 6. (a) Proportion of soil moisture sites exceeding estimated field capacity as a function of mean soil moisture, plotted by season (offset for clarity). (b) Mean catchment drainage as a function of mean catchment soil moisture, plotted by season. Dotted line shows range of hidden points from Spring season. 85x43mm (300 x 300 DPI)