Constraining Dynamic TOPMODEL Responses using Fuzzy Rule Based Performance Measures of Uncertain Water Table Information

Environmental Science Research
LANCASTER
Hydrology and Fluid Dynamics Group

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INTRODUCTION

Dynamic TOPMODEL, a conceptual rainfall-runoff model, is applied to the Maimai M8 catchment (3.8 ha, figure 1a), New Zealand using rainfall-runoff and uncertain water table (W_t) information in the analysis of model performance. Different parametric representations of hillslope (HS) and valley bottom (VB) hydrological similarity units (HSU's) were used to improve the spatial representation of the Dynamic TOPMODEL structure (figure 1b). The continuous time series W_t information is obtained from multiple tensiometric observations from both near stream and hillslope locations (figure 1c,d). Fuzzy estimates of W_t dynamics for each time step were derived from the variability in the observed data (see figure 2). Parameter interactions b/t the two HSU's are assessed using Monte Carlo simulations. Conclusions are drawn as to the usefulness of uncertain (fuzzy) information in evaluating Dynamic TOPMODEL's structure and in constraining model parameters (figures 5-7).

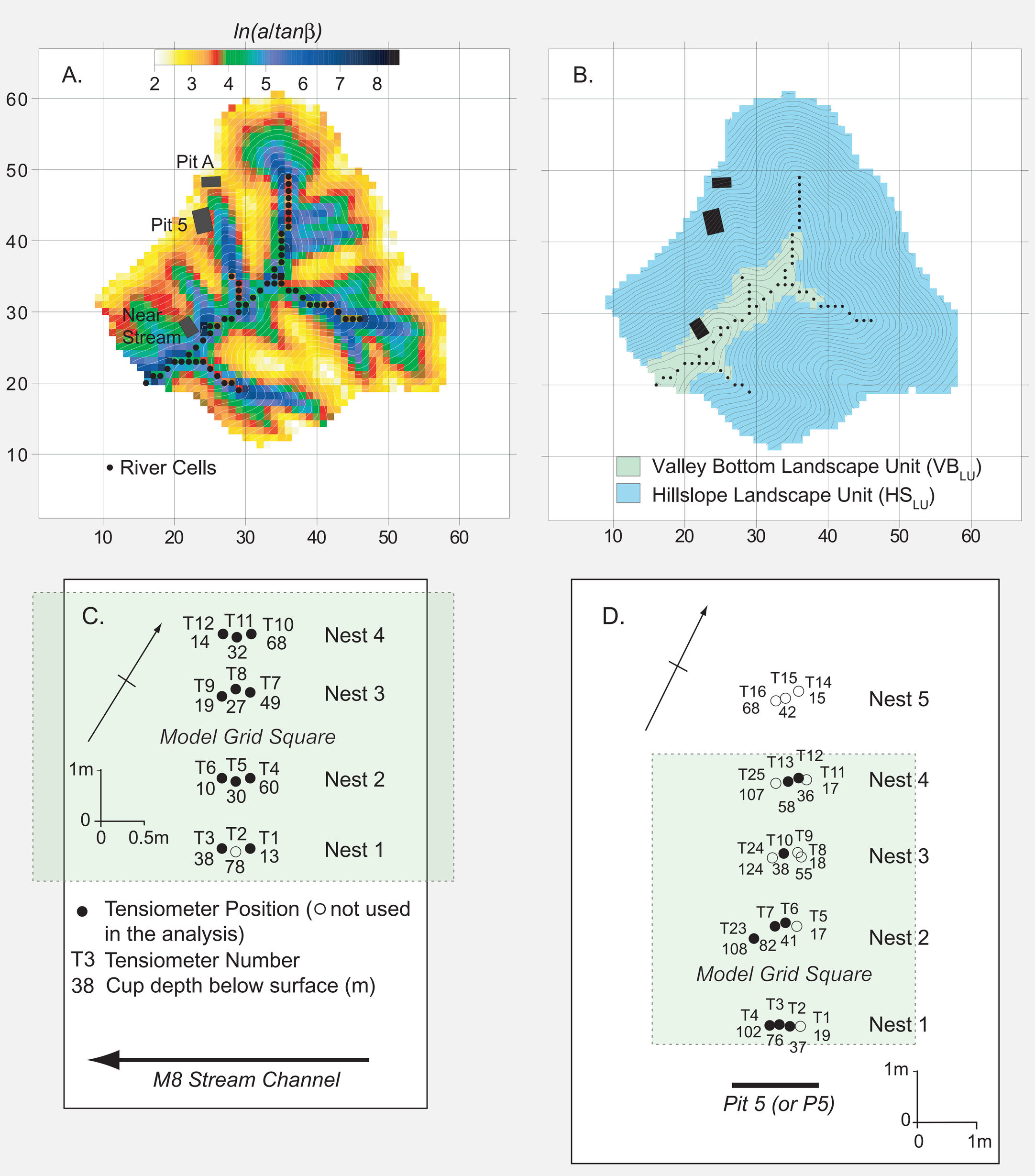


Figure 1: Maimai M8 catchment: The spatial variability of (A) the ln(a/tanβ) index and (B) the VB and HS LU's. Details of tensiometer instrumentation at (C) the Near Stream and (D) the Pit 5 sites (see McDonnell, [1990] and McGlynn et al. [2002] for description of data/catchment).

MAIN AIMS - to explore:

How fuzzy rules can be applied to imperfect and imprecise knowledge that is at a scale consistant with the effective model gridscale

To challenge the assumptions of the Dynamic TOPMODEL structure at Maimai using multi-response observations

The assessment model performance using Monte Carlo simulations within the Generalised Likelihood Uncertainty Estimation (GLUE) procedure [*Beven and Binley*, 1991].

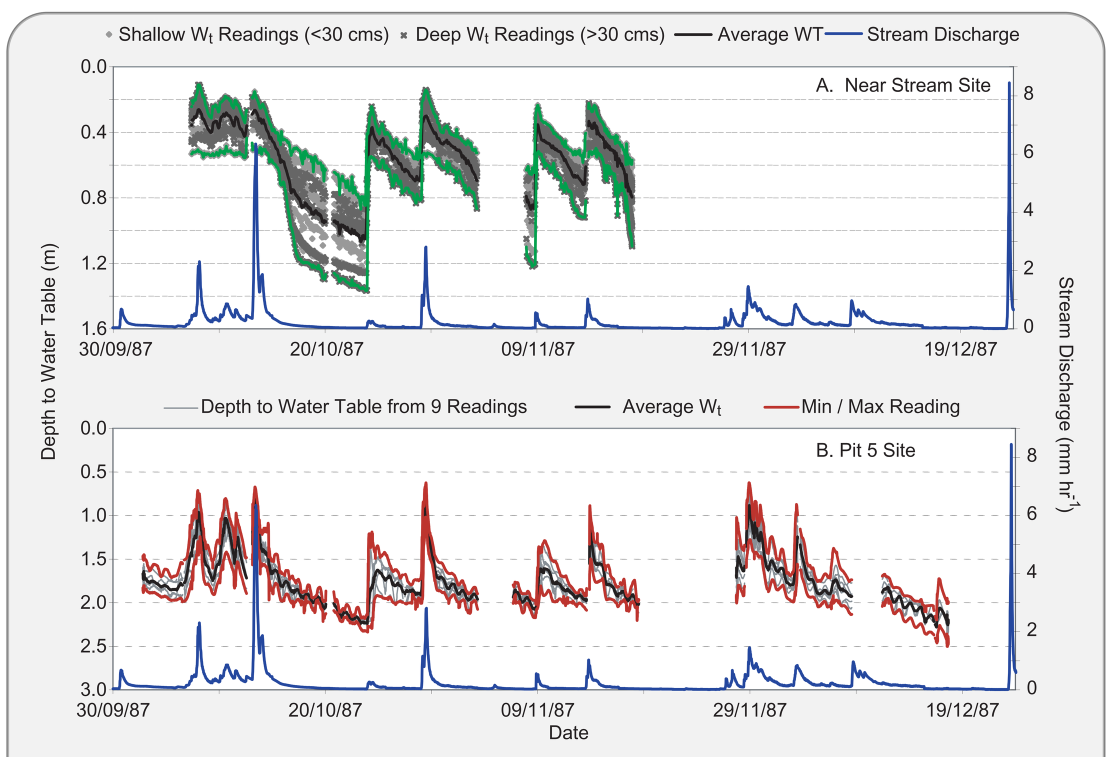


Figure 2: Observed water table responses calculated from the tensiometer data for both (A) Near Stream and (B) Pit 5 tensiometer sites. The plot shows the resultant upper and lower min and max limits for the water table responses defining the model gridscale variability of the observations.

Dynamic TOPMODEL

Dynamic TOPMODEL [*Beven and Freer*, 2001] is a new version of TOPMODEL that relaxes some of the assumptions of the original form [*Beven and Kirkby*, 1979]. This new formulation allows for local accounting of hydrological fluxes and storages (see figure 3), relaxing the quasi steady state assumption of a water table parallel to the local surface slope expressed through the derivation of the ln(a/tanβ) index. The dynamics of the subsurface saturated zone during wetting and drying event periods can now be simulated. The increased flexibility of the model structure allows for the spatial definition of different HSU's, each potentially having different functional forms and parameterisations. Transfers between HSU's are calculated using a kinematic wave approximation.

Parameter	Units	Lower Limits*	Upper Limits*	Description
SZM	[m]	0.001 {0.005}	0.012 {0.017}	Form of the exponential decline in conductivity
ln(T0)	[m^2 hr^{-1}]	7.0 {-7.0}	3.0 {3.0}	Effective lateral saturated transmissivity
SR_{max}	[m]	0.005 {0.005}	0.08 {0.08}	Maximum soil root zone deficit
SR_{ini}	[m]	0.00 {0.00}	0.01 {0.01}	Initial root zone deficit
CHV	[m hr^{-1}]	250 {250}	1500 {1500}	Channel routing velocity
T_d	[hr]	0.10 {0.10}	40.0 {40.0}	Unsaturated zone time delay
		0.05 {0.01}	0.60 {0.30}	Effective porosity
S_{max}	[m]	0.60 {0.60}	2.00 {2.00}	Maximum effective deficit of the subsurface storage zone

*Parameter upper and lower ranges for both the *valley bottom* and *hillslope* (in {}'s)

Table 1: Parameter ranges for the VB_{LU} and the HS_{LU} for the Monte-Carlo simulations

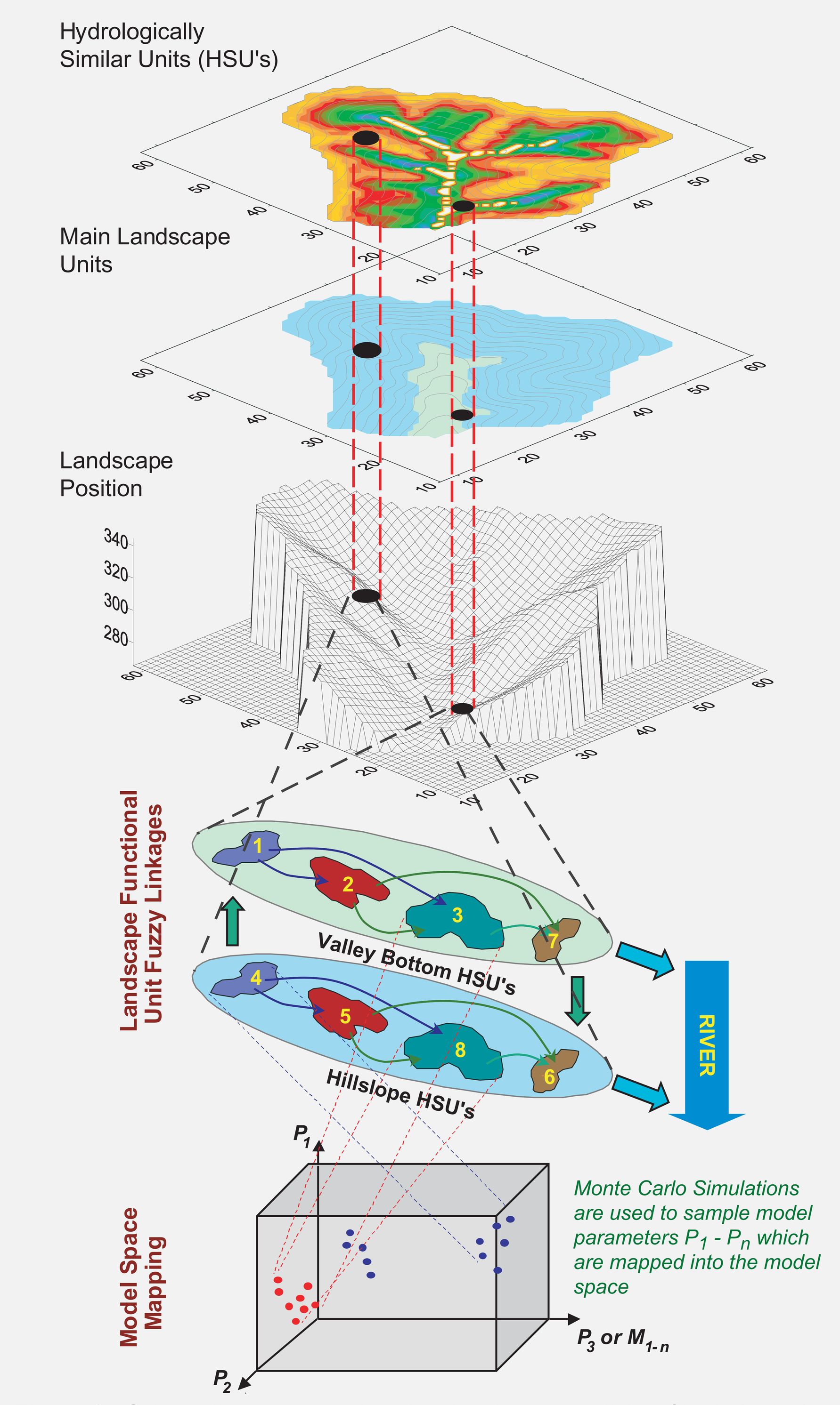


Figure 3: Schematic representation showing the connectivity of hydrological response units in the new Dynamic TOPMODEL. The catchment is grouped according to landscape units that have distinct hydrological functional forms. Kinematic routing of both sub-landscape units (HSU's in this case using basic topographic characteristics) and between landscape units is determined by multi-directional downslope fluxes from digital terrain analysis.

Methods

Fuzzy Water Table Measures

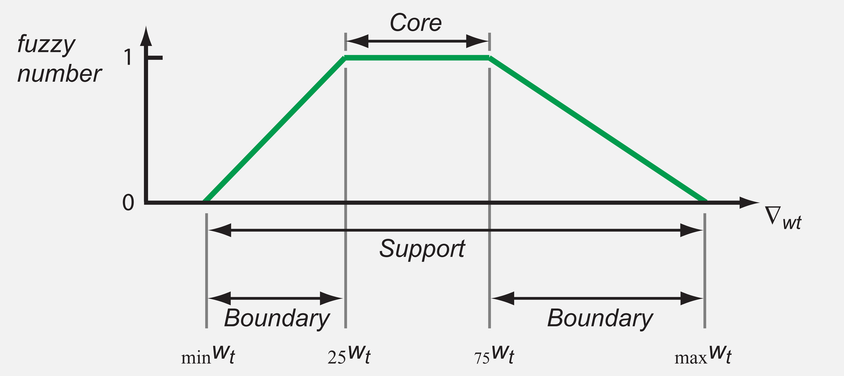
For each tensiometer nest location, and within an area equivalent to an effective model gridscale, a number of tensiometer readings were available (figure 1). Using this information a distribution of W_t elevations for each time step at each location was calculated (figure 2). Relationships between -ve tension and height above W_t were derived for periods with a known W_t evelation [*Freer*, 1998]. The distribution of water table elevations was used to derive fuzzy estimates of the water table depth for the whole time series that explicitly includes the temporal variability of the uncertainty in the observations (figure 4). These data were used to further constrain the spatial representation of the model having previously conditioned the model using the rainfall-runoff data.

Dynamic TOPMODEL GLUE Simulations

For each simulation run all parameters listed in Table 1 were randomly assigned a value appropriate to the ranges specified for each LU. A uniform sampling strategy of the parameter ranges was deployed to express the lack of knowledge of the expected distribution and covariance of the parameter values. The model streamflow and W_t predictions were compared to the observed data using an appropriate Performance Measure (PM - for streamflow this was R^2), non-behavioural simulations were rejected.

Results

Table 2 lists the number of behavioural simulations from 2 sets of Monte Carlo simulation runs (run_2 had more constrained parameter ranges). The efficiency of the sampling is noted. The relationships between parameters for the final set of behavioural simulations (i.e. that were classed as behavioural for all PM's) are shown in figure 5. Figure 6 shows the range of uncertainty in the model predictions for each observed data series. Finally, figure 7 highlights the variability in the model dynamics from behavioural parameter sets obtained using different PM's by using summary model responses.



In Equation Form the Fuzzy Performancs Measure for all timesteps for the WT simulations can be shown as

$$L\left[M(\Theta|Y_T, W_T)\right]=\sum_{t=1}^{n}\begin{cases}0 & z_t \geq {}_{max}w_t\\ \dfrac{z_t - {}_{75}w_t}{{}_{max}w_t - {}_{75}w_t} & {}_{75}W_t \leq z_t \leq {}_{max}W_t\\ 1 \quad \cdots\textit{if}\cdots & {}_{25}W_t \leq z_t \leq {}_{75}W_t\\ \dfrac{z_t - {}_{min}w_t}{{}_{25}w_t - {}_{min}w_t} & {}_{min}W_t \leq z_t \leq {}_{25}W_t\\ 0 & z_t \leq {}_{min}w_t\end{cases}$$

Where $M(\Theta|Y_t, W_t)$ indicates the ith model, conditioned on input data Y_t and observations W_t. Where W_t is the observed and Z_t is the simulated water table at time $_t$. The observed W_t limits ($_{min}W_t$, $_{25}W_t$, $_{75}W_t$ and $_{max}W_t$) are determined at each time step from the apparent variability in the W_t observations at the model gridscale (see figure 2).

Figure 4: An example of the construction and terminology of a Fuzzy Performance Measure applied to the Water Table Information at the NS and Pit 5 Sites.

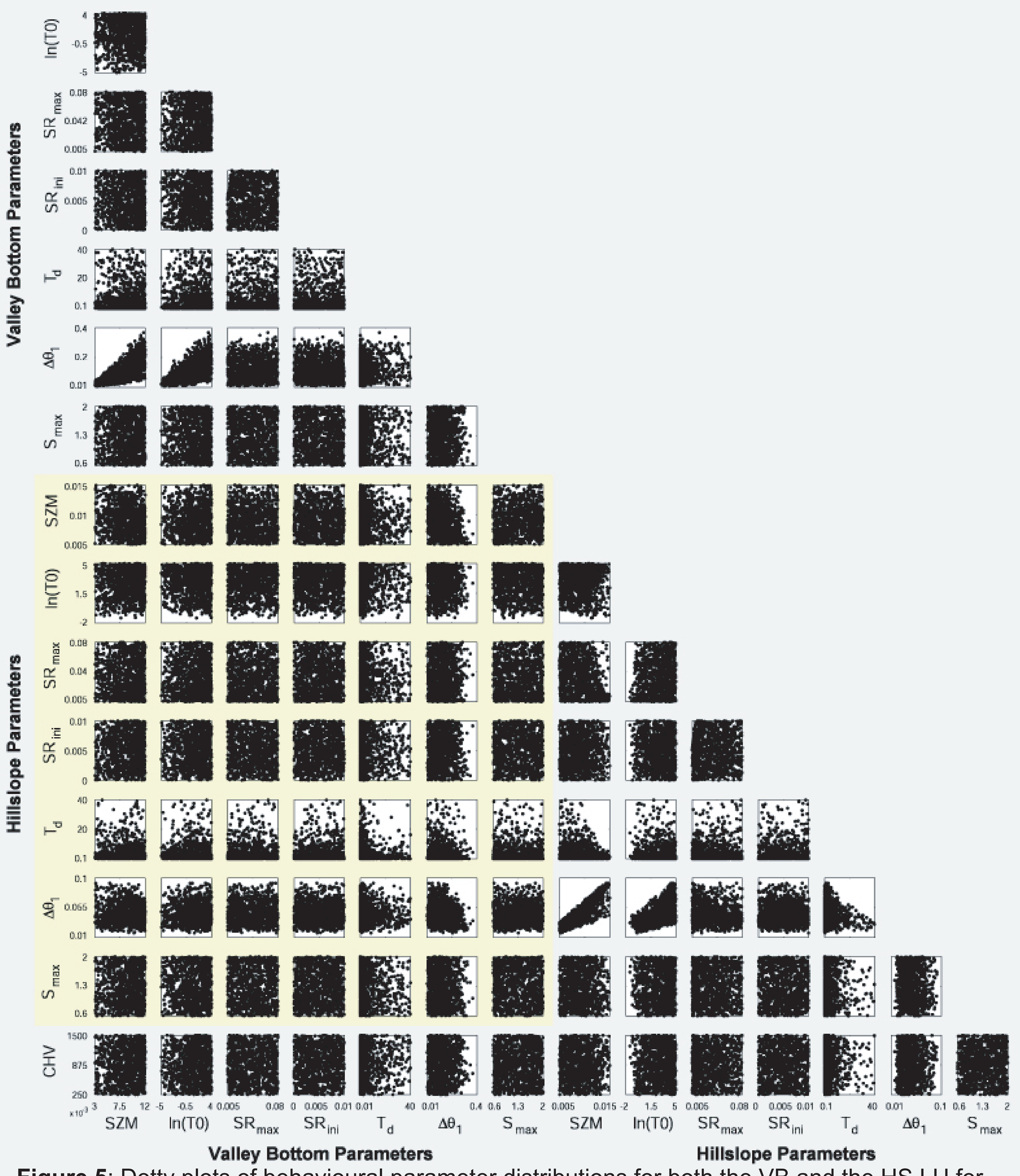


Figure 5: Dotty plots of behavioural parameter distributions for both the VB and the HS LU for parameter sets that were classed as behavioural for all three performance measures. The main matrix of dotty plots shows the correlation between pairs of parameters within the same LU and between the HS and VB LU's (the yellowed area).

	run_1 behavioural simulations*		run_2 behavioural simulations**		run_1 and run_2
Acceptability Criteria	**Total Number**	**Sampling Efficiency (%)**	**Total Number**	**Sampling Efficiency (%)**	**Sampling Efficiency Increase**
Discharge only	2,327,664	41.56	1,016,325	84.69	2.0
NS_{wt} *only*	196,591	3.51	118,519	9.87	2.8
$P5_{wt}$ *only*	16,195	0.28	39,128	3.26	11.5
Discharge and NS_{wt}	84,636	1.51	98,218	8.18	5.4
Discharge and $P5_{wt}$	11,987	0.21	34,205	2.80	13.3
NS_{wt} *and* $P5_{wt}$	614	0.011	3,692	0.31	28.2
Discharge, NS and $P5_{wt}$	419	0.007	3,184	0.26	37.1

* Total number of all simulations was 5,600,000 ** Total number of all simulations was 1,200,000

Table 2: Behavioural simulations for individual and combined acceptance criteria for the performance measures identified in Table 3 from both run_1 and run_2.

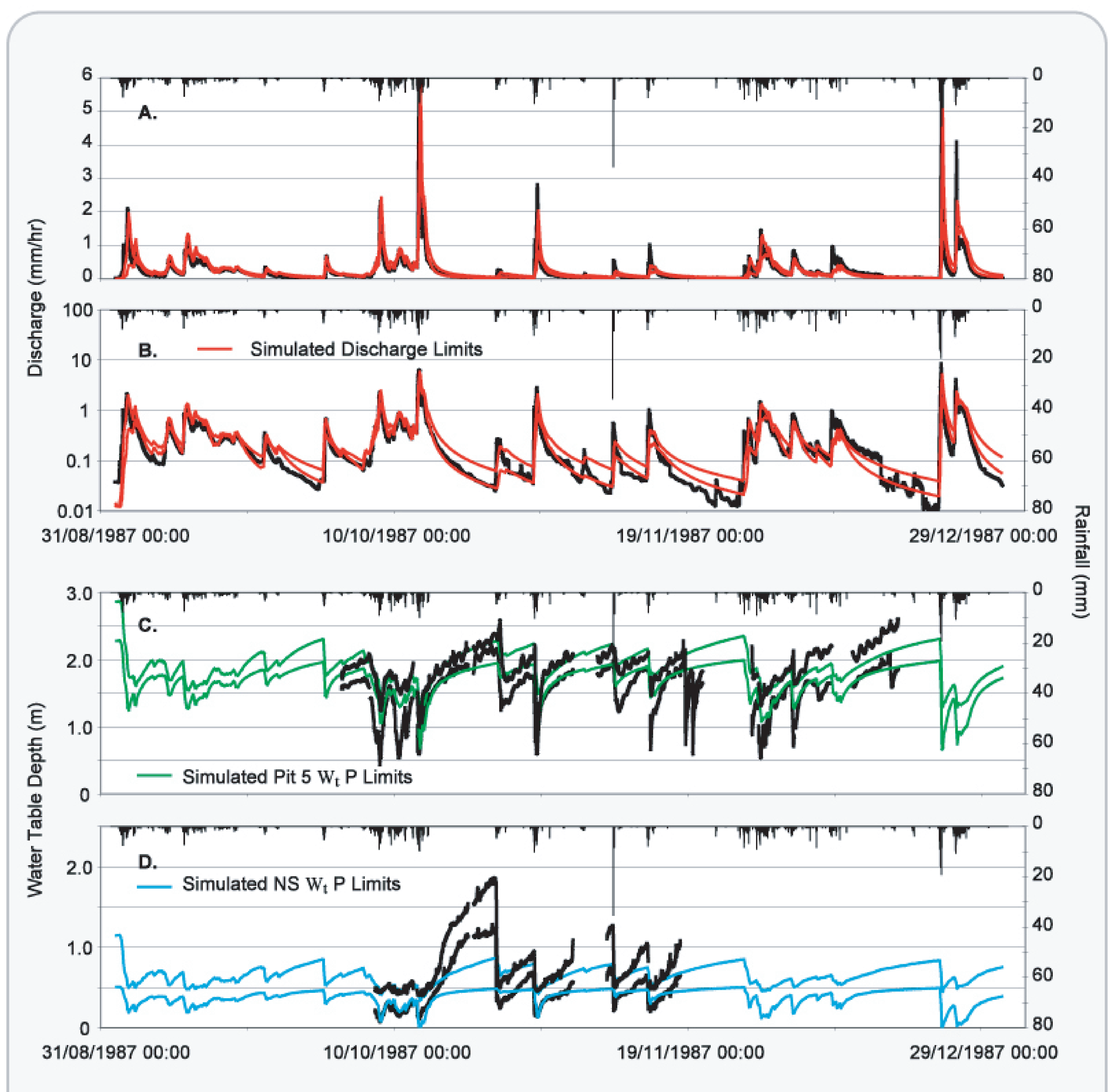


Figure 6: Final GLUE Simulations using PM's updated from Discharge, NS W_t and Pit 5 W_t data showing behavioural possibility bounds for a) Discharge, b) ln(Discharge), C) Pit 5 W_t and D) NS W_t simulations.

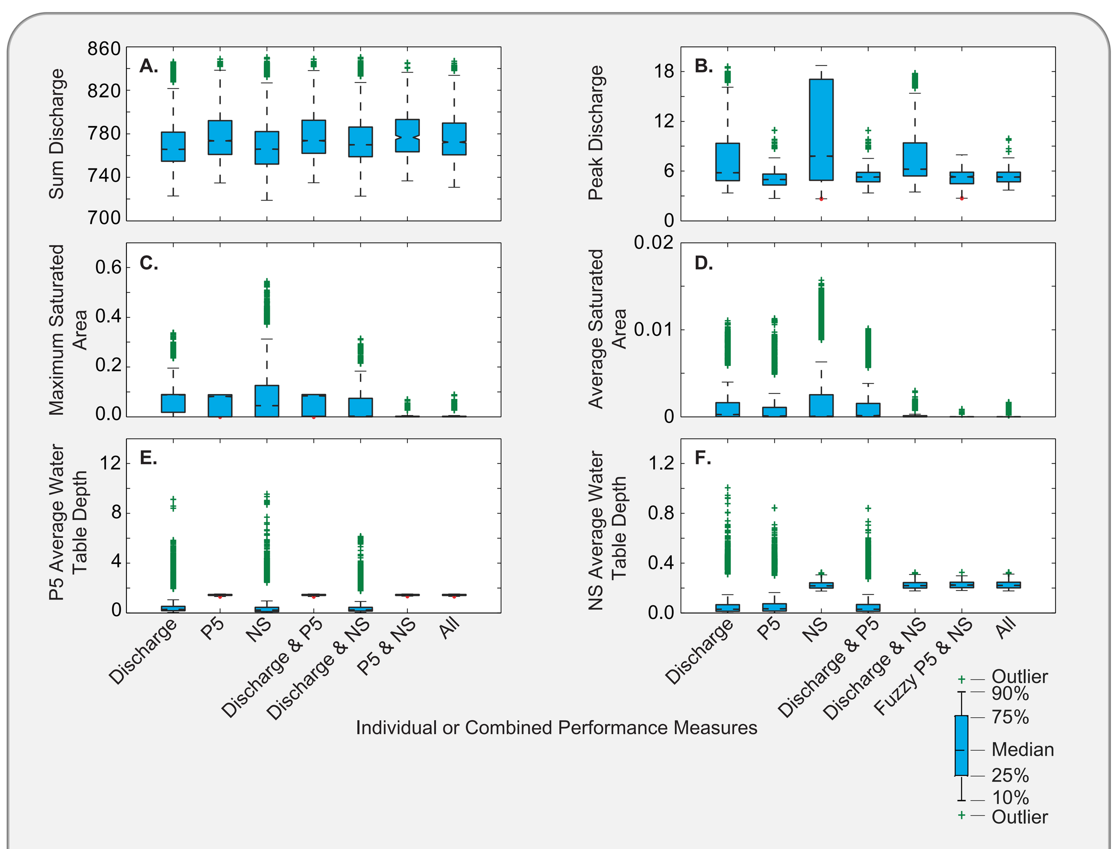


Figure 7: Distributions of summary model responses for behavioural simulations using different PM's or combinations of PM's

Conclusions

- Possible uncertainties in data should be assessed, especially when point scale spatial information is used to assess preditions made at the model gridscale.
- Fuzzy numbers are a useful way to define PM's for observational uncertainties that are time variant and have complex error structures.
- Although behavioural simulations are retained prediction limits are not always bracketed by the observational uncertainties. Importantly, is this model structural error or unaccounted for observational error ?
- Equifinality of the final behavioural parameter sets suggests the need for an uncertainty analysis procedure such as GLUE

Acknowledgements

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Poster Papers

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Related Papers

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